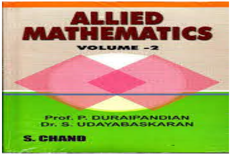
CAMA 15C : Mathematics - I



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Algebraic equation

Let us consider

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n.$

This a polynomial in x of degree n provided $a_0 \neq 0$.

The equation is obtained by putting $f(x) = 0$ is called an

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Fundamental Theorem

1. Every polynomial equation $f(x) = 0$ has at least one root real or complex.
2. Every polynomial equation $f(x) = 0$ of n^{th} degree has n roots and only n roots.

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Types of Problems

1. Relation between the roots and coefficient of equations.
2. Imaginary roots and irrational roots.
3. Transformation of equations.
4. Reciprocal equations.
5. Newton's method.

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2.1. Relation between the roots and coefficient of equations

Given equation be

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0.$

Divide the equation by a_0, then

$$x^n + \frac{a_1}{a_0}x^{n-1} + \frac{a_2}{a_0}x^{n-2} + \ldots + \frac{a_{n-1}}{a_0}x + \frac{a_n}{a_0} = 0.$$

i.e., $x^n + p_1x^{n-1} + p_2x^{n-2} + \ldots + p_{n-1}x + p_n = 0.$ (say $p_i = \frac{a_i}{a_0}$)

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Let $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$, be its roots.

Then, we have

$$S_1 = \sum \alpha_1 = -p_1$$

$$S_2 = \sum \alpha_1 \alpha_2 = p_2$$

$$S_3 = \sum \alpha_1 \alpha_2 \alpha_3 = -p_3$$

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$$S_n = \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_n = (-1)^n p_n$$

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Problem 1.

If α and β are the roots of $2x^2 + 3x + 5 = 0$, then

find $\alpha + \beta, \alpha\beta$.

Solution.

Given $2x^2 + 3x + 5 = 0$.

$x^2 + \frac{3}{2}x + \frac{5}{2} = 0.$

Here $p_1 = \frac{3}{2}$ and $p_2 = \frac{5}{2}$

We know that $\alpha + \beta = -p_1 = -\frac{3}{2}$ and

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If α , β and γ are the roots of $ax^3 + bx^2 + cx + d = 0$, then

find $\alpha + \beta + \gamma, \ \alpha\beta + \beta\gamma + \gamma\alpha, \ \alpha\beta\gamma.$

Solution.

Given $ax^3 + bx^2 + cx + d = 0.$

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$$

We know that $\alpha + \beta + \gamma = -p_1 = -b/a$

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2.2 Imaginary roots and irrational roots

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Problem 1. Form the equation, one of whose root is $\sqrt{3} + \sqrt{5}$

Solution.

Given $\sqrt{3} + \sqrt{5}$ is one of whose root of the required equation.

The other root of the same equation are

$$-\sqrt{3} + \sqrt{5}, \quad \sqrt{3} - \sqrt{5}, \text{ and } -\sqrt{3} - \sqrt{5}$$

Therefore, the required equation is

$$[x - (\sqrt{3} + \sqrt{5})][x - (\sqrt{3} - \sqrt{5})][x - (-\sqrt{3} + \sqrt{5})][x - (-\sqrt{3} - \sqrt{5})] = 0$$

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$$[x - (\sqrt{3} + \sqrt{5})][x - (\sqrt{3} - \sqrt{5})][x - (-\sqrt{3} + \sqrt{5})][x - (-\sqrt{3} - \sqrt{5})] = 0$$

Problems.

Problem 1. Form the equation, one of whose root is $\sqrt{3}+\sqrt{5}$

Solution.

Given $\sqrt{3}+\sqrt{5}$ is one of whose root of the required equation.

The other root of the same equation are

$$-\sqrt{3}+\sqrt{5}, \quad \sqrt{3}-\sqrt{5}, \text{ and } -\sqrt{3}-\sqrt{5}$$

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$$[x-(\sqrt{3}+\sqrt{5})][x-(\sqrt{3}-\sqrt{5})][x-(-\sqrt{3}+\sqrt{5})][x-(-\sqrt{3}-\sqrt{5})]=0$$

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$$(\quad (a+b)(a-b) = a^2-b^2 \quad)$$

$$(x^2-2x\sqrt{3}+3-5)\ (x^2+2x\sqrt{3}+3-5)=0$$

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$$(x^2 - 2x\sqrt{3} + 3 - 5)\ (x^2 + 2x\sqrt{3} + 3 - 5) = 0$$

$$(x^2 - 2x\sqrt{3} - 2)\ (x^2 + 2x\sqrt{3} - 2) = 0$$

$$[(x^2 - 2) - 2x\sqrt{3}]\ [(x^2 - 2) + 2x\sqrt{3}] = 0$$

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Problem 2. Solve $x^4 - 12x - 5 = 0$ given $-1 + 2i$ is a root.

Solution.

Since $-1 + 2i$ is a root, $-1 - 2i$ will also be a root of the equation.

The factor corresponding to the two roots is

$(x + 1 - 2i)\,(x + 1 + 2i) = 0$

$(x + 1)^2 - (2i)^2 = 0$

$x^2 + 2x + 1 + 4 = 0$

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$x^2 + 2x + 5 \,=\, 0$

Now divide $x^4 - 12x - 5 = 0$ by $x^2 + 2x + 5 = 0$

$$
\begin{array}{r|l}
 & x^2 - 2x - 1 \\
\hline
x^2 + 2x + 5 & x^4 \qquad\qquad - 12x - 5 \\
 & -x^4 - 2x^3 - 5x^2 \\
\hline
 & -2x^3 - 5x^2 - 12x \\
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 & \qquad x^2 \quad + 2x + 5 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
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$\therefore$ The other roots are given by

$$x^2 - 2x - 1 = 0 \qquad (ax^2 + bx + c = 0)$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} \qquad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = 1 \pm \sqrt{2}$$

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Problem 3. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ given that

$2 + i\sqrt{3}$ is a root.

Solution.

Since $2 + i\sqrt{3}$ is a root, $2 - i\sqrt{3}$ will also be a root of the equation.

The factor corresponding to the two roots is

$(x - 2 - i\sqrt{3})\ (x - 2 + i\sqrt{3})\ =\ 0$

$(x - 2)^2 - (i\sqrt{3})^2\ =\ 0$

$x^2 - 4x + 4 + 3\ =\ 0$

$x^2 - 4x + 7\ =\ 0$

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Now divide $x^4 - 4x^2 + 8x + 35 = 0$ by $x^2 - 4x + 7 = 0$

$$\begin{array}{r|l}
 & x^2 + 4x + 5 \\
\hline
x^2 - 4x + 7 & x^4 - 4x^2 + 8x + 35 \\
 & -x^4 + 4x^3 - 7x^2 \\
\hline
 & 4x^3 - 11x^2 + 8x \\
 & -4x^3 + 16x^2 - 28x \\
\hline
 & 5x^2 - 20x + 35 \\
 & -5x^2 + 20x - 35 \\
\hline
 & 0
\end{array}$$

Now divide $x^4 - 4x^2 + 8x + 35 = 0$ by $x^2 - 4x + 7 \ = \ 0$

$$
\begin{array}{r|lllll}
 & & & x^2 & +4x & +5 \\
\hline
x^2 - 4x + 7 & x^4 & & -4x^2 & +8x & +35 \\
 & -x^4 & +4x^3 & -7x^2 & & \\
\cline{2-4}
 & & 4x^3 & -11x^2 & +8x & \\
 & & -4x^3 & +16x^2 & -28x & \\
\cline{3-5}
 & & & 5x^2 & -20x & +35 \\
 & & & -5x^2 & +20x & -35 \\
\cline{4-6}
 & & & & & 0
\end{array}
$$

$\therefore$ The other roots are given by

$$x^2 + 4x + 5 = 0 \qquad (ax^2 + bx + c = 0)$$

$$x = \frac{-4 \pm \sqrt{16 - 20}}{2} \qquad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$x = \frac{-4 \pm 2i}{2}$$

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$$x = \frac{-4 \pm \sqrt{16 - 20}}{2} \qquad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$x = \frac{-4 \pm 2i}{2}$$

$$x = -2 \pm i$$

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Problem 4. Solve the equation $x^4 - 11x^2 + 2x + 12 = 0$ given that

$\sqrt{5} - 1$ is a root.

Solution.

Since $\sqrt{5} - 1$ is a root, $-\sqrt{5} - 1$ will also be a root of the equation.

The factor corresponding to the two roots is

$$(x + 1 - \sqrt{5})\,(x + 1 + \sqrt{5}) = 0$$

$$(x + 1)^2 - (\sqrt{5})^2 = 0$$

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Now divide $x^4 - 11x^2 + 2x + 12 = 0$ by $x^2 + 2x - 4 = 0$

$$
\begin{array}{r|rrrrr}
 & & x^2 & -2x & -3 & \\
\hline
x^2+2x-4 & x^4 & & -11x^2 & +2x & +12 \\
 & -x^4 & -2x^3 & +4x^2 & & \\
\hline
 & & -2x^3 & -7x^2 & +2x & \\
 & & 2x^3 & +4x^2 & -8x & \\
\hline
 & & & -3x^2 & -6x & +12 \\
 & & & 3x^2 & +6x & -12 \\
\hline
 & & & & & 0
\end{array}
$$

Now divide $x^4 - 11x^2 + 2x + 12 = 0$ by $x^2 + 2x - 4 = 0$

$$
\begin{array}{r|l}
 & x^2 - 2x - 3 \\
\hline
x^2 + 2x - 4 & x^4 \qquad - 11x^2 + 2x + 12 \\
 & -x^4 - 2x^3 + 4x^2 \\
\hline
 & -2x^3 - 7x^2 + 2x \\
 & 2x^3 + 4x^2 - 8x \\
\hline
 & -3x^2 - 6x + 12 \\
 & 3x^2 + 6x - 12 \\
\hline
 & 0
\end{array}
$$

$\therefore$ The other roots are given by

$$x^2 - 2x - 3 = 0 \qquad (x^2 - (\alpha+\beta)x + \alpha\beta = 0)$$

$$(x-3)(x+1) = 0 \qquad (\alpha\beta = c;\ \alpha+\beta = -b)$$

$$x = -1, 3 \qquad (\alpha = -1;\ \beta = 3)$$

$\therefore$ The roots are $\pm\sqrt{5} - 1$, -1, 3.

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Problems.

5. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that $1 - \sqrt{5}$ is a root.

6. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1 + \sqrt{-1}$ is a root.

7. Solve the equation $x^4 + 2x^2 - 16x + 77 = 0$ given that $-2 + \sqrt{-7}$ is a root.

8. Solve the equation $3x^5 - 4x^4 - 42x^3 + 52x^2 + 27x - 36 = 0$ given that $\sqrt{2} + \sqrt{5}$ is a root.