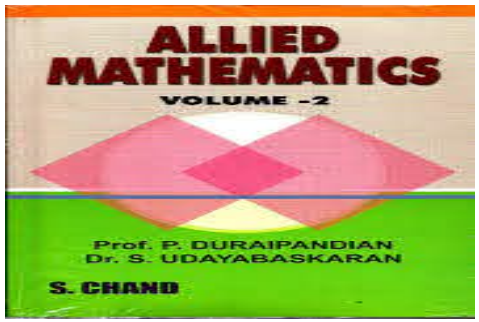
Allied Mathematics - I
Unit-II Theory of Equation



Dr S. Srinivasan

Assistant Professor,
Department of Mathematics ,
Periyar Arts College,
Cuddalore,
Tamil nadu.

Email: smrail@gmail.com
Cell: 7010939424

Types of Problems

1. Relation between the roots and coefficient of equations.
2. Imaginary roots and irrational roots.
3. Transformation of equations.
4. Reciprocal equations.
5. Newton's method.

2.3 Transformation of Equation by decreasing or increasing roots by a constant

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In this method, the simplification can easily find the solution of the given equation.

Problem 1.

Diminish by 3 the roots of the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$.

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Diminish by 3.

	1	-4	-18	-3	2
3		3	-3	-63	-198
	1	-1	-21	-66	-196

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	1	-4	-18	-3	2
3		3	-3	-63	-198
	1	-1	-21	-66	-196

$$
\begin{array}{c|cccc}
 & 1 & -1 & -21 & -66 \\
3 & & 3 & 6 & -45 \\
\hline
 & 1 & 2 & -15 & -111
\end{array}
$$

$$
\begin{array}{c|cccc}
 & 1 & -1 & -21 & -66 \\
3 & & 3 & 6 & -45 \\
\hline
 & 1 & 2 & -15 & -111
\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & 2 & -15 \\
3 & & 3 & 15 \\
\hline
 & 1 & 5 & 0
\end{array}
$$

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\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & 2 & -15 \\
3 & & 3 & 15 \\
\hline
 & 1 & 5 & 0
\end{array}
$$

$$
\begin{array}{c|cc}
 & 1 & 5 \\
3 & & 3 \\
\hline
 & 1 & 8
\end{array}
$$

$$
\begin{array}{c|cccc}
 & 1 & -1 & -21 & -66 \\
3 & & 3 & 6 & -45 \\
\hline
 & 1 & 2 & -15 & -111
\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & 2 & -15 \\
3 & & 3 & 15 \\
\hline
 & 1 & 5 & 0
\end{array}
$$

$$
\begin{array}{c|cc}
 & 1 & 5 \\
3 & & 3 \\
\hline
 & 1 & 8
\end{array}
$$

$\therefore$ the required equation is $x^4 + 8x^3 - 111x - 196 = 0$.

Problem 2.

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Diminish by 2.

	1	-1	-10	4	24
2		2	2	-16	-24
	1	1	-8	-12	0

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Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Diminish by 2.

	1	-1	-10	4	24
2		2	2	-16	-24
	1	1	-8	-12	0

$$
\begin{array}{c|cccc}
 & 1 & 1 & -8 & -12 \\
2 & & 2 & 6 & -4 \\
\hline
 & 1 & 3 & -2 & -16
\end{array}
$$

		1	1	-8	-12
2			2	6	-4
		1	3	-2	-16

	1	3	-2
2		2	10
	1	5	8

$$
\begin{array}{c|cccc}
 & 1 & 1 & -8 & -12 \\
2 & & 2 & 6 & -4 \\
\hline
 & 1 & 3 & -2 & -16
\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & 3 & -2 \\
2 & & 2 & 10 \\
\hline
 & 1 & 5 & 8
\end{array}
$$

$$
\begin{array}{c|cc}
 & 1 & 5 \\
2 & & 2 \\
\hline
 & 1 & 7
\end{array}
$$

$$
\begin{array}{c|cccc}
 & 1 & 1 & -8 & -12 \\
2 & & 2 & 6 & -4 \\
\hline
 & 1 & 3 & -2 & -16
\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & 3 & -2 \\
2 & & 2 & 10 \\
\hline
 & 1 & 5 & 8
\end{array}
$$

$$
\begin{array}{c|cc}
 & 1 & 5 \\
2 & & 2 \\
\hline
 & 1 & 7
\end{array}
$$

$\therefore$ the resulting equation is $x^4 + 7x^3 + 8x^2 - 16x = 0$.

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

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$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

		1	7	8	−16
1			1	8	16
		1	8	16	0

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

		1	7	8	-16
1			1	8	16
		1	8	16	0

$$\Rightarrow x^2+8x+16=0$$

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

		1	7	8	−16
1			1	8	16
		1	8	16	0

$$\Rightarrow x^2+8x+16=0$$

$$\Rightarrow (x+4)(x+4)=0$$

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

	1	7	8	-16
1		1	8	16
	1	8	16	0

$$\Rightarrow x^2+8x+16=0$$

$$\Rightarrow (x+4)(x+4)=0$$

$$\Rightarrow x=-4,-4$$

$$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$$

$$\Rightarrow x=0 \text{ and } x^3+7x^2+8x-16=0$$

	1	7	8	-16
1		1	8	16
	1	8	16	0

$$\Rightarrow x^2+8x+16=0$$

$$\Rightarrow (x+4)(x+4)=0$$

$$\Rightarrow x=-4,-4$$

$$\Rightarrow x=-4,\ -4,\ 0,\ 1$$

$\Rightarrow x\left(x^3+7x^2+8x-16\right)=0$

$\Rightarrow x=0$ and $x^3+7x^2+8x-16=0$

	1	7	8	−16
1		1	8	16
	1	8	16	0

$\Rightarrow x^2+8x+16=0$

$\Rightarrow (x+4)(x+4)=0$

$\Rightarrow x=-4,-4$

$\Rightarrow x=-4,-4,0,1$

$\therefore$ the roots of the given equation is $x=-2,-2,2,3$.

Problem 3.

Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1

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Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1

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Diminish by 1.

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Solution.

Given equation is $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$.

Diminish by 1.

	1	-4	-7	22	24
1		1	-3	-10	12
	1	-3	-10	12	36

Problem 3.

Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1

and hence solve the equation.

Solution.

Given equation is $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$.

Diminish by 1.

	1	-4	-7	22	24
1		1	-3	-10	12
	1	-3	-10	12	36

$$
\begin{array}{c|cccc}
 & 1 & -3 & -10 & 12 \\
1 & & 1 & -2 & -12 \\
\hline
 & 1 & -2 & -12 & 0
\end{array}
$$

$$
\begin{array}{c|rrr}
 & 1 & -3 & -10 & 12 \\
1 & & 1 & -2 & -12 \\
\hline
 & 1 & -2 & -12 & 0
\end{array}
$$

$$
\begin{array}{c|rrr}
 & 1 & -2 & -12 \\
1 & & 1 & -1 \\
\hline
 & 1 & -1 & -13
\end{array}
$$

$$
\begin{array}{c|cccc}
 & 1 & -3 & -10 & 12 \\
1 & & 1 & -2 & -12 \\
\hline
 & 1 & -2 & -12 & 0
\end{array}
$$

$$
\begin{array}{c|ccc}
 & 1 & -2 & -12 \\
1 & & 1 & -1 \\
\hline
 & 1 & -1 & -13
\end{array}
$$

$$
\begin{array}{c|cc}
 & 1 & -1 \\
1 & & 1 \\
\hline
 & 1 & 0
\end{array}
$$

$\therefore$ the resulting equation is $x^4 - 13x^2 + 36 = 0$.

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Put $x^2 = y$

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$y^2 - 13y + 36 = 0$.

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Put $x^2 = y$

$y^2 - 13y + 36 = 0.$

$(y - 9)(y - 4) = 0.$

$\therefore$ the resulting equation is $x^4 - 13x^2 + 36 = 0$.

Put $x^2 = y$

$y^2 - 13y + 36 = 0.$

$(y - 9)(y - 4) = 0.$

$y = 9, 4.$

$\therefore$ the resulting equation is $x^4 - 13x^2 + 36 = 0$.

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$y^2 - 13y + 36 = 0$.

$(y - 9)(y - 4) = 0$.

$y = 9, 4$.

i.e., $x^2 = 4, 9$.

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$y^2 - 13y + 36 = 0$.

$(y - 9)(y - 4) = 0$.

$y = 9, 4$.

i.e., $x^2 = 4, 9$.

$\Rightarrow x = -2, 2, -3, 3$

$\therefore$ the resulting equation is $x^4 - 13x^2 + 36 = 0$.

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$y^2 - 13y + 36 = 0$.

$(y - 9)(y - 4) = 0$.

$y = 9, 4$.

i.e., $x^2 = 4, 9$.

$\Rightarrow x = -2, 2, -3, 3$

$\therefore$ the roots of the given equation is $x = 0, 4, -1, 5$

Problem 4.

Increase the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ by 7

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Given equation is $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Increase by 7.

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Given equation is $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Increase by 7.

i.e., diminish by -7.

Problem 4.

Increase the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ by 7

Solution.

Given equation is $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Increase by 7.

i.e., diminish by -7.

	3	7	-15	1	-2
-7		-21	98	-581	4060
	3	-14	83	-580	4058

$$
\begin{array}{r|rrrr}
 & 3 & -14 & 83 & -580 \\
-7 & & -21 & 245 & -2296 \\
\hline
 & 3 & -35 & 328 & -2876
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 3 & -14 & 83 & -580 \\
-7 & & -21 & 245 & -2296 \\
\hline
 & 3 & -35 & 328 & -2876
\end{array}
$$

$$
\begin{array}{r|rrr}
 & 3 & -35 & 328 \\
-7 & & -21 & 392 \\
\hline
 & 3 & -56 & 720
\end{array}
$$

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 & 3 & -14 & 83 & -580 \\
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$$
\begin{array}{r|rr}
 & 3 & -56 \\
-7 & & -21 \\
\hline
 & 3 & -77
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 3 & -14 & 83 & -580 \\
-7 & & -21 & 245 & -2296 \\
\hline
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$$

$$
\begin{array}{r|rrr}
 & 3 & -35 & 328 \\
-7 & & -21 & 392 \\
\hline
 & 3 & -56 & 720
\end{array}
$$

$$
\begin{array}{r|rr}
 & 3 & -56 \\
-7 & & -21 \\
\hline
 & 3 & -77
\end{array}
$$

$\therefore$ the resulting equation is $3x^4 - 77x^3 + 720x^2 - 2876x + 4058 = 0$.

Problem 5.

Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

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Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

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Solution.

Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Problem 5.

Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

and hence solve the equation.

Solution.

Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Increase by 2.

Problem 5.

Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

and hence solve the equation.

Solution.

Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Increase by 2.

i.e., diminish by -2.

Problem 5.

Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

and hence solve the equation.

Solution.

Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Increase by 2.

i.e., diminish by -2.

		1	-1	-10	4	24
-2			-2	6	8	-24
		1	-3	-4	12	0

Problem 5.

Increase the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2

and hence solve the equation.

Solution.

Given equation is $x^4 - x^3 - 10x^2 + 4x + 24 = 0$.

Increase by 2.

i.e., diminish by -2.

		1	-1	-10	4	24
-2			-2	6	8	-24
		1	-3	-4	12	0

$$
\begin{array}{r|rrrr}
 & 1 & -3 & -4 & 12 \\
-2 & & -2 & 10 & -12 \\
\hline
 & 1 & -5 & 6 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 1 & -3 & -4 & 12 \\
-2 & & -2 & 10 & -12 \\
\hline
 & 1 & -5 & 6 & 0
\end{array}
$$

$$
\begin{array}{r|rrr}
 & 1 & -5 & 6 \\
-2 & & -2 & 14 \\
\hline
 & 1 & -7 & 20
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 1 & -3 & -4 & 12 \\
-2 & & -2 & 10 & -12 \\
\hline
 & 1 & -5 & 6 & 0
\end{array}
$$

$$
\begin{array}{r|rrr}
 & 1 & -5 & 6 \\
-2 & & -2 & 14 \\
\hline
 & 1 & -7 & 20
\end{array}
$$

$$
\begin{array}{r|rr}
 & 1 & -7 \\
-2 & & -2 \\
\hline
 & 1 & -9
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 1 & -3 & -4 & 12 \\
-2 & & -2 & 10 & -12 \\
\hline
 & 1 & -5 & 6 & 0
\end{array}
$$

$$
\begin{array}{r|rrr}
 & 1 & -5 & 6 \\
-2 & & -2 & 14 \\
\hline
 & 1 & -7 & 20
\end{array}
$$

$$
\begin{array}{r|rr}
 & 1 & -7 \\
-2 & & -2 \\
\hline
 & 1 & -9
\end{array}
$$

$\therefore$ the resulting equation is $x^4 - 9x^3 + 20x^2 = 0$.

$$\Rightarrow x^2\left(x^2-9x+20\right)=0$$

$$\Rightarrow x^2\left(x^2-9x+20\right)=0$$

$$\Rightarrow x^2=0 \text{ and } x^2-9x+20=0$$

$\Rightarrow x^2(x^2 - 9x + 20) = 0$

$\Rightarrow x^2 = 0$ and $x^2 - 9x + 20 = 0$

Consider $x^2 - 9x + 20 = 0$.

$\Rightarrow x^2(x^2 - 9x + 20) = 0$

$\Rightarrow x^2 = 0$ and $x^2 - 9x + 20 = 0$

Consider $x^2 - 9x + 20 = 0$.

$(x - 5)(x - 4) = 0$.

$\Rightarrow x^2(x^2 - 9x + 20) = 0$

$\Rightarrow x^2 = 0$ and $x^2 - 9x + 20 = 0$

Consider $x^2 - 9x + 20 = 0$.

$(x - 5)(x - 4) = 0$.

$x = 5, 4$.

$\Rightarrow x^2(x^2 - 9x + 20) = 0$

$\Rightarrow x^2 = 0$ and $x^2 - 9x + 20 = 0$

Consider $x^2 - 9x + 20 = 0$.

$(x - 5)(x - 4) = 0$.

$x = 5, 4$.

$\Rightarrow x = 0, 0, 4, 5$

$\Rightarrow x^2(x^2 - 9x + 20) = 0$

$\Rightarrow x^2 = 0$ and $x^2 - 9x + 20 = 0$

Consider $x^2 - 9x + 20 = 0$.

$(x - 5)(x - 4) = 0$.

$x = 5, 4$.

$\Rightarrow x = 0, 0, 4, 5$

$\therefore$ the roots of the given equation is $x = -2, -2, 2, 3$

Problems

Problem 6.

Diminish the roots of the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ by 2

Problem 7.

Increase the roots of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ by 2

Problem 8.

Solve the equation $x^4 - 12x^3 + 48x^2 + 72x + 35 = 0$ by diminishing

the roots so that the second term is absent.