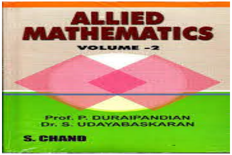
Allied Mathematics - I
Unit-II Theory of Equation



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Types of Problems

1. Relation between the roots and coefficient of equations.
2. Imaginary roots and irrational roots.
3. Transformation of equations.
4. Reciprocal equations.
5. Newton's method.

2.5 Newton's method

Suppose that we want to approximate the solution to $f(x) = 0$.

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Suppose that we have somehow found an initial approximation to this solution say, x_0.

This initial approximation is probably not all that good, so we can find a better approximation.

This is easy enough to do.

First, we will get the tangent line to $f(x)$ at x_0.

$$y = f(x_0) + f'(x_0)(x - x_0)$$

Now, take a look at the graph below.

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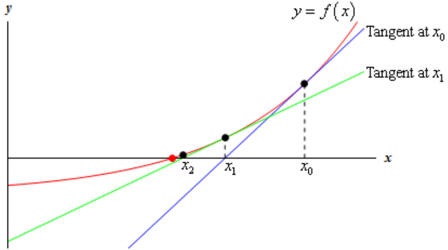

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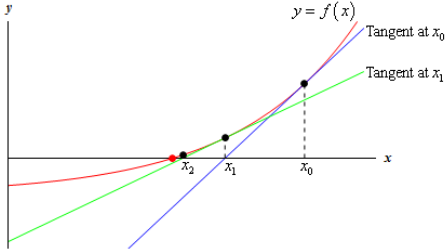

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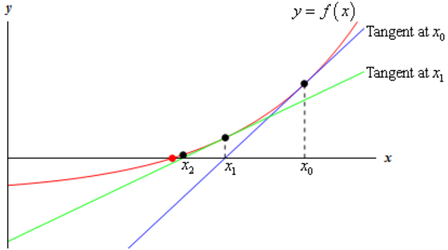

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Let us call this point where the tangent at x_0 crosses the X-axis by x_1.

We will use this point as our new approximation to the solution.

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Substitute in the tangent line and solve for x_1 as follows.

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$$0 = f(x_0) + f'(x_0)(x_1 - x_0)$$

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$$0 = f(x_0) + f'(x_0)(x_1 - x_0)$$

$$(x_1 - x_0) = -\frac{f(x_0)}{f'(x_0)}$$

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x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)}
\end{aligned}
$$

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So, we can find the new approximation provided the derivative is not zero

at the original approximation.

Now we repeat the whole process to find an even better approximation.

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This should lead to the question of when do we stop?

How many times do we go through this process?

One of the more common stopping points in the process is to continue

until two successive approximations agree to a given number of decimal places.

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3. Continue the iterative process using the formula until the root

   is found to the desired accuracy.

Problem 1.

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$\therefore$ the roots lies between $[1, 2]$.

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1^{st} approximation: if $n = 0$

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

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$$x_1 = x_0 - \frac{f(1)}{f'(1)}$$

$$x_1 = 1 - \frac{(1^2 - 3)}{2}$$

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$$x_1 = 2$$

2^{nd} approximation: if $n = 1$

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$$x_2 = 1.75$$

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$$x_3 = 1.75 - \frac{f(1.75)}{f'(1.75)}$$

$$x_3 = 1.75 - \frac{((1.75)^2 - 3)}{2 \times 1.75}$$

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$$x_3 = 1.732143$$

4^{th} approximation: if $n = 3$

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Thus, the approximate root of $x^2 - 3$ is

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Thus, the approximate root of $x^2 - 3$ is1.732050.

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Let $f(x) = x^2 - 12$.

Put $x = 1 \Rightarrow f(1) = 1 - 12 = -ve$

$x = 2 \Rightarrow f(2) = 4 - 12 = -ve$

$x = 3 \Rightarrow f(2) = 9 - 12 = -ve$

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$\therefore$ the roots lies between $[3, 4]$.

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{f(3)}{f'(3)}$$

$$x_1 = 3 - \frac{(3^2 - 12)}{2.3}$$

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$$x_1 = 3 - \frac{f(3)}{f'(3)}$$

$$x_1 = 3 - \frac{(3^2 - 12)}{2.3}$$

$$x_1 = 3 + \frac{3}{6}$$

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$$x_2 = 3.5 - \frac{((3.5)^2 - 12)}{2 \times 3.5}$$

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$$x_2 = 3.5 - \frac{f(3.5)}{f'(3.5)}$$

$$x_2 = 3.5 - \frac{((3.5)^2 - 12)}{2 \times 3.5}$$

$$x_2 = 3.4642$$

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$$x_3 = 3.4642 - \frac{f(3.4642)}{f'(3.4642)}$$

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$$x_3 = 3.4641$$

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Thus, the approximate root of $x^2 - 12$ is 3.4641.

Problems

Find by Newton's method an approximate value of the

positive root of the equations.

(i) $x^3 - 2x - 5 = 0$

(ii) $x^3 - 3x + 1 = 0$

(iii) $x^3 - 5x + 3 = 0$